

# Non-interacting $KN$ contribution in the QCD sum rule for the pentaquark $\Theta^+(1540)$

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## Abstract

We perform a QCD sum rule analysis for the pentaquark baryon  $\Theta^+$  with the non-interacting  $KN$  contribution treated carefully. The coupling of the  $\Theta^+$  current to the  $KN$  state is evaluated by applying the soft kaon theorem and vacuum saturation. When using a five-quark current including scalar and pseudo-scalar diquarks, the  $KN$  contribution turns out not to be very important and the previous result of the negative parity  $\Theta^+$  is reproduced again. The Borel analysis of the correlation function for  $\Theta^+$  with the  $KN$  continuum states subtracted yields the mass of the  $J^P = 1/2^-$   $\Theta^+$  around 1.5 GeV.

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## I. INTRODUCTION

The experiment by the LEPS Collaboration [1] at SPring-8 suggested the existence of the exotic baryon  $\Theta^+(1540)$  of strangeness  $S = +1$ . Subsequent experiments at CLAS collaboration as well as the reanalyses of the existing data seem to support the LEPS result. Yet, recently many negative results have been also reported [2]. We need more convincing experiments with high statistics especially near the threshold regions [3].

A quantitative analysis was first performed for exotic baryons including  $\Theta^+$  by Diakonov et. al. using the chiral soliton model [4]. They predicted a small mass around 1.5 GeV and a narrow width of order of ten MeV for  $\Theta^+$ . In the chiral soliton model,  $J^P = 1/2^+$  has been also predicted, with its flavor state being in the antidecuplet representation  $\bar{10}$  of SU(3). Contrary, in a quark model, a five quark state with positive parity requires additional orbital excitation and therefore, the low mass around 1.5 GeV seems difficult to explain [5, 6, 7]. Furthermore, lattice QCD [8, 9, 10] and QCD sum rule [11, 12] analyses seem to support a negative parity state. In view of variation of theoretical predictions, dynamics of the low energy QCD is less understood than we have thought previously.

One of the difficulties in the theoretical methods for the exotic pentaquark states is that they can couple to scattering meson-baryon states. In the lattice QCD or QCD sum rule analyses where two point correlation functions are studied, the five-quark currents for  $\Theta^+$  couples to the  $KN$  scattering state which would contaminate the analysis of the resonance of  $\Theta^+$ . This is related to the fact that there are only three color degrees of freedom, and therefore hadronic operators of more than four quarks must always couple to two (or more) color singlet hadronic states. Therefore, the removal of the  $KN$  state is a key issue to understand the properties of  $\Theta^+$ .

Motivated by such an observation, Kondo, Morimatsu and Nishikawa proposed a method to isolate the  $KN$  continuum contribution in their QCD sum rule analysis by Fierz rearranging the five quark operator into a sum of products of two color-singlet components [13]. Then they expressed the non-interacting  $KN$  contribution as a convolution of the kaon and nucleon correlation functions. In their analysis, the  $KN$  contribution was large and changed the parity of  $\Theta^+$  before and after the subtraction of the  $KN$  contribution. However, in their convolution integral, the momenta for the kaon and nucleon correlation functions were not always in the asymptotic region where the OPE can be applicable.

Recently, two of the present authors (S.H. Lee and Y. Kwon) and H. Kim performed a QCD sum rule analysis where they attempted to isolate the  $KN$  contribution by applying the soft kaon theorem [14]. Their method is reliable concerning the application of the OPE, since it contains an evaluation of a single correlation function which is well defined in the asymptotic region. It was then found that the  $KN$  contribution is negligibly small and the original observation by Sugiyama, Doi and Oka remained unchanged. In their analysis a coupling of a five quark current to the nucleon state was considered. Since the coupling of the ground state nucleon to the five-quark current is expected to be small, it naturally suppressed the relevant  $KN$  contribution. Intuitively, however, the  $\Theta^+$  current contains three quark terms which is coupled by the ground state nucleon. Diagrammatically, these two processes are depicted in Fig.1. Naively, one expects that the process of Fig.1-(b) would be a major coupling to the  $KN$  state.

In this report, we study the  $KN$  scattering contribution in the QCD sum rule. We apply the previously proposed method of soft kaon theorem to compute the  $KN$  to the vacuum matrix element of the pentaquark current. This corresponds to the background method

when computing meson-baryon couplings in the QCD sum rule. After taking a commutation relation of the pentaquark current with the kaonic axial charge, we find an expression containing the scalar operator  $\langle qq \rangle$ . This will be replaced by the vacuum condensate by assuming the vacuum saturation. Such a process can be understood in an effective meson theory in terms of the matrix element of the axial vector current for the kaon decay.

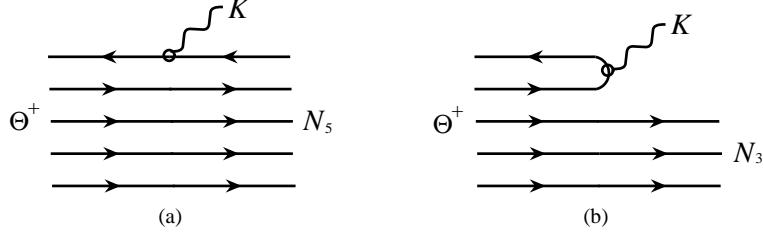


FIG. 1: (a) five quark nucleon decay (b) fall apart decay

## II. $KN$ CONTRIBUTION TO THE SUM RULE

Let us start with the  $KN$  contribution in the two point correlation function

$$\Pi(q) = \int \frac{d^4x}{(2\pi)^4} e^{iq \cdot x} \langle 0 | T[J_\Theta(x) \bar{J}_\Theta(0)] | 0 \rangle. \quad (1)$$

On the right hand side, the  $KN$  scattering state is generated by the coupling

$$\langle 0 | J_\Theta | KN \rangle \equiv i \lambda_{KN} \gamma_5 u_N(p), \quad (2)$$

where  $u_N(p)$  is a nucleon spinor;

$$\Pi^{KN}(q) = i |\lambda_{KN}|^2 \int \frac{d^4p}{(2\pi)^4} \frac{\gamma_5(p + m_N) \gamma_5}{p^2 - m_N^2} \frac{1}{(p - q)^2 - m_K^2}. \quad (3)$$

Hence the determination of the coupling strength  $\lambda_{KN}$  is the crucial thing.

To this end, we apply the soft kaon theorem to the l.h.s. of Eq. (2)

$$\langle 0 | J_\Theta | KN \rangle = -\frac{1}{f_K} \langle 0 | [Q_5^K, J_\Theta] | N \rangle, \quad (4)$$

where  $Q_5^K = \int d^3x \bar{s} \gamma_0 \gamma_5 u$  is the axial charge with kaon flavor and  $f_K$  is the kaon decay constant,  $f_K \sim 100$  MeV. In Ref. [14], the commutation relation on the right hand side was computed, leading to the matrix element  $\langle 0 | J_{N_5} | N \rangle$  where  $J_{N_5}$  is a nucleon current written in terms of five quarks.

Now consider the matrix element  $\langle 0 | J_\Theta | KN \rangle$  in a hadronic description. We assume that the pentaquark current has a component of  $KN$ :

$$J_\Theta = a KN + \dots, \quad (5)$$

where  $a$  is a constant, and  $K$  and  $N$  are kaon and nucleon fields. In terms of quarks,  $N$  is written as a three quark current. If chiral symmetry is spontaneously broken, the axial

charge has a piece of time derivative of the Nambu-Goldstone boson field. In particular we have

$$Q_5^K = -if_K \int d^3x \partial_0 K + \dots \quad (6)$$

Applying the canonical commutation relation  $[\Pi_K(x), K(y)] = i\delta^3(x - y)$  where  $\Pi_K(x) = \partial_0 K(x)$ , we find

$$[Q_5^K, J_\Theta] = af_K N + \dots \quad (7)$$

In this procedure, the kaon field in (5) has been replaced by the decay constant  $f_K \sim \langle \bar{q}q \rangle$ , which is a consequence of the spontaneous breaking of chiral symmetry. In this way we expect to have a nucleon current written by three quarks as in the r.h.s of (7).

In the QCD sum rule calculation, the contributions corresponding to (7) can be computed first extracting the  $(\bar{s}q)_K(qqq)_N$  piece by the Fierz rearrangement of  $J_\Theta(\bar{s}qqq)$ , and then the  $\bar{q}q$  pair in  $[Q_5^K, (\bar{s}q)_K]$  is replaced by the vacuum expectation value. Let us now consider the following  $\Theta^+$  current

$$J_\Theta = \epsilon^{abc} \epsilon^{def} \epsilon^{cfg} (u_a^T C d_b) (u_d^T C \gamma_5 d_e) C \bar{s}_g^T \quad (8)$$

as used in Ref. [12], where  $C$  is a charge conjugation matrix and  $T$  denotes transpose. As shown in Ref. [14], we obtain

$$[Q_5^K, J_\Theta] = J_{N_5}, \quad (9)$$

where

$$\begin{aligned} J_{N_5} = & \epsilon^{abc} \epsilon^{def} \epsilon^{cfg} [\{u_a^T C \gamma_5 s_b\} \{u_d^T C \gamma_5 d_e\} C \bar{s}_g^T \\ & + \{u_a^T C d_b\} \{u_d^T C s_e\} C \bar{s}_g^T + \{u_a^T C d_b\} \{u_d^T C \gamma_5 d_e\} C \gamma_5 \bar{d}_g^T]. \end{aligned} \quad (10)$$

Forming a scalar quantity  $\bar{q}q$  in (10) after the Fierz rearrangement and replace it by the vacuum expectation value (vacuum saturation) we obtain

$$\begin{aligned} \langle 0 | J_\Theta | KN \rangle &= -\frac{1}{f_K} \langle 0 | [Q_5^K, J_\Theta] | N \rangle \\ &= -\frac{1}{6f_K} \langle \bar{s}s + \bar{d}d \rangle \langle 0 | J_{N_3} | N \rangle \\ &= -\frac{f_K m_K^2}{6m_s} \langle 0 | J_{N_3} | N \rangle, \end{aligned} \quad (11)$$

where the three-quark nucleon current is given by

$$J_{N_3} = \epsilon^{abc} \{ (u_a^T C d_b) u_c - (u_a^T C \gamma_5 d_b) \gamma_5 u_c \}. \quad (12)$$

In deriving (11), we have used the Gell-Mann-Oakes-Renner relation:  $f_K^2 m_K^2 = m_s \langle 0 | \bar{s}s + \bar{d}d | 0 \rangle$ , where  $m_s$  is the current mass of the strange quark. It is interesting to see that in Eq. (12), we find the Ioffe's current which has the optimal coupling to the physical nucleon state. Defining the coupling to the three-quark current by

$$\langle 0 | J_{N_3} | N \rangle = i\lambda_{N_3} u_N(q) \quad (13)$$

and compare (11) with (2), we find

$$\lambda_{KN_3} = -\frac{f_K m_K^2}{6m_s} \lambda_{N_3}. \quad (14)$$

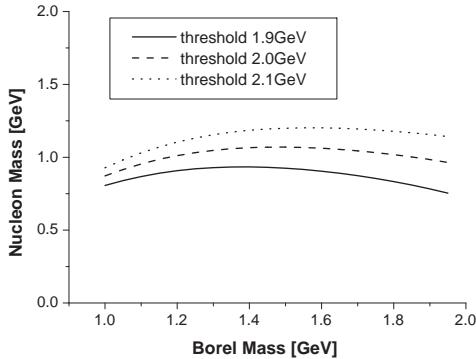


FIG. 2: The masses of the nucleon in the QCD sum rule as functions of the Borel mass in units of GeV. Three different values 1.9, 2.0 and 2.1 GeV are used for the threshold value.

In order to determine the coupling  $\lambda_{N_3}$ , we have performed the sum rule analysis for the nucleon in an old fashioned manner with the positive parity states are properly projected out by using the projector  $(1 + \gamma_0)/2$ . Fig. 2 shows the Borel mass analysis for the sum rule for nucleon. Here we have verified that the Borel stability is well achieved. From this, we find the mass of the nucleon around 1GeV when a threshold value  $\sqrt{s_0} \sim 2.0$ GeV is used. Furthermore, we find that the coupling strength  $|\lambda_{N_3}|^2 \simeq 1.6 \times 10^{-4}$ GeV<sup>6</sup>. Using this value together with  $f_K \sim 100$  MeV and  $m_s \sim 130$  MeV, we obtain  $|\lambda_{KN_3}|^2 \simeq 14 \times 10^{-8}$ GeV<sup>10</sup>, which is larger than the value obtained in Ref. [14] by about a factor ten, where the coupling of the five-quark current  $J_{N_5}$  was employed. At this point, we have verified that the contribution from the three-quark nucleon component is dominant in the coupling  $\Theta^+ \rightarrow KN$ .

### III. REANALYSIS OF THE SUM RULE FOR $\Theta^+$

The sum rule for  $\Theta^+$  originally performed by Sugiyama, Doi and Oka is now modified by subtracting the  $KN$  contribution. It reads

$$|\lambda_{\Theta\pm}|^2 e^{-\frac{m_\Theta^2}{M^2}} = \int_0^{\sqrt{s_0}} dq_0 e^{-\frac{q_0^2}{M^2}} \rho_{OPE}^\pm(q_0) - \int_{m_K+m_N}^\infty dq_0 e^{-\frac{q_0^2}{M^2}} \rho_{KN}^\pm(q_0), \quad (15)$$

where  $\rho_{OPE}^\pm$  is given in Ref. [12]. Because the l.h.s. is positive definite, the reliable sum rule should have the r.h.s. with the positive sign also. Various contributions of the r.h.s. of Eq. (15) are plotted in Figs. 3 for the positive (right panel) and negative (left panel) parity cases using a threshold value  $\sqrt{s_0} = 2.2$  GeV. In principle, there remains a question of the convergence of the OPE. Here we have included, as before, the results up to dimension 6. Furthermore we have adopted a threshold value  $\sqrt{s_0} = 2.2$  GeV, slightly larger than the previously used one around 1.8 GeV [12]. The larger value seems consistent with the nucleon sum rule here. Using the larger threshold values, we find that the sum rule for the negative parity case is qualitatively similar to the previous result [12], while the spectral function of positive parity state turns its sign from negative (unphysical) to positive (physical). The

change in the sign of the positive parity case, however, does not necessarily mean the existence of a resonance state. It indicates that the sum rule for the positive parity is rather sensitive to (or equivalently unstable against) the choice of the threshold value. For the  $KN$  contribution, we can see from Figs. 3, that it is substantial for the negative parity case, while almost negligible for the positive parity case. The less important contribution of the positive parity may be understood due to the suppression factor of the p-wave coupling of the  $KN$  state to  $\Theta^+$  [15].

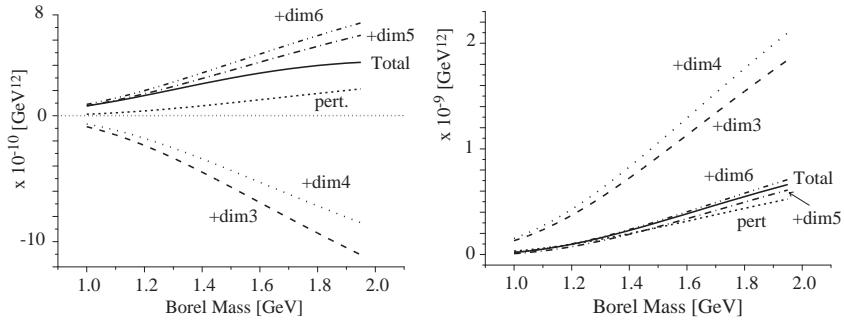


FIG. 3: Borel curves for the negative parity (left panel) and positive parity (right panel) pentaquarks as we add up the OPE order by order. In the total result (solid line), the  $KN$  contribution is subtracted.

Now, we have performed the sum rule analysis for  $\Theta^+$ . In Fig. 4, we have shown the Borel mass dependence of the mass of  $\Theta^+$  for the negative parity case for three threshold values of  $\sqrt{s_0} = 2.2, 2.3$  and  $2.4$  GeV. Using the larger threshold values, we have found stable result as shown in Fig. 4. The resulting mass of  $\Theta^+$  turns out to be around  $1.5 - 1.6$  GeV. Although not shown here, when  $\sqrt{s_0} = 1.8$  GeV is used, we found unstable result. Hence, we have found once again a negative parity  $\Theta^+$  at the mass region around where experimentally expected. On the contrary, for the positive parity case, we could not find a realistic solution for the sum rule equation, though the l.h.s. of Eq. (15) is positive. As anticipated, this is due to the instability of the spectral function, which should not be the case when there is a stable (or quasi-stable) state.

#### IV. SUMMARY AND CONCLUSIONS

In this paper we have reanalyzed the QCD sum rule for  $\Theta^+$  with the  $KN$  continuum contribution being subtracted out. Physically such a contribution should be dominated by the coupling of the  $\Theta^+$  current to the kaon and three-quark nucleon state, which we were able to estimate by utilizing the soft kaon theorem and vacuum saturation.

The  $KN$  contribution was found to be substantial for the negative parity but almost negligible for the positive parity case. Furthermore, we should point out that the spectral function of OPE was rather stable against the threshold parameter for the negative parity case, but extremely sensitive for the positive parity case. These results have once again lead to the  $\Theta^+$  of negative parity with a mass around  $1.5-1.6$  GeV, while it was not possible to find a solution in the positive parity state.

Although the QCD sum rule is a systematic method based on the established technique of QCD, there remain still several questions, such as convergence of the OPE and dependence

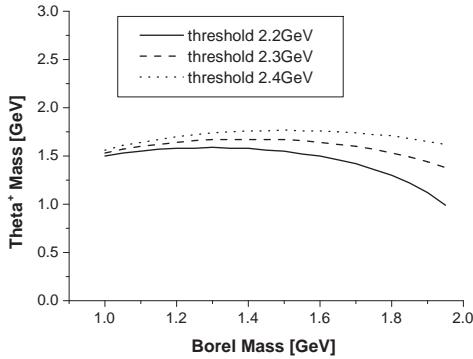


FIG. 4: The masses of  $\Theta^+$  in the QCD sum rule as functions of the Borel mass in units of GeV. Three different values 2.2, 2.3 and 2.4 GeV are used for the threshold value.

of the choice of pentaquark currents. Together with the separation of the  $KN$  contribution, these questions must be very important. Further study with proper consideration of them is needed for better understanding of the exotic states.

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